Resonance

The Missing Phenomenon in Hemodynamics

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To simulate a short segment of the aorta, we studied wave propagation in an elastic tube with a side branch balloon. The small balloon simulated the organ (group of arterioles). Ligation of this side branch would reduce the moduli of the higher harmonics when the length of the side branch was appropriate. Electrical analogy of vessels was used to analyze this phenomenon. This simulation can explain the ligation results we found in rats. It may also clarify the discrepancies between the prediction of the Womersley equation and the experimental results. We suggest that the aorta and the closely attached organ can produce coupled oscillation; theoretically, this structure is equivalent to a resonance circuit. (*Circulation Research* 1991;69:246–249)

The pressure wave in an artery is accepted as the summation of an incident pressure wave and a series of reflected waves originating from peripheral systems.[1,2] However, several phenomena in hemodynamics, such as the amplification of the high-frequency components of the pressure wave in some large arteries and the radically different shapes of the flow and pressure pulses in the ascending aorta, cannot be fully understood by current theories.

In addition, measurements conducted by Milnor and Bertram,[3] Atabek et al,[4] Ling et al,[5] and many others found that the ratio of the reactive and resistive terms cannot be fitted by linearized equations such as the Womersley equations. All of these linearized equations greatly underestimate resistance for the small Womersley number α ($\alpha^2=R^2\omega\rho/\eta$, where R is the radius of the vessel, ω is the angular frequency, ρ is the density of the fluid, and η is the viscosity coefficient).

Because all linearized models had essentially the same defect, there have been suggestions that the inconsistency arises from omission of the nonlinear terms in the Navier-Stokes equations.[6]

We have performed some experiments in rats in vivo, and the results cannot be explained by current theory either.[7–9] When we temporarily clamped the left renal artery, the harmonic moduli of the pressure wave taken at the tail artery were significantly reduced. The moduli for components above and including the second harmonic fell by about 40% or more. Superior mesenteric arterial ligation created an entirely different profile; in this case, significant increases, ranging from 17% to 28%, began at the third harmonic. Ligation of the splenic artery caused a significant effect from the third harmonic and above, with decreasing moduli.

In this report, we present an additional linear phenomenon—resonance. It may solve these problems; since resonance effectively brings in a capacitance in the impedance and greatly changes the ratio of the reactive and resistive parts, it would significantly reduce the high-frequency impedance and facilitate the blood pressure wave propagation. It would also help the blood to enter the organ. Therefore, because of the resonance effect, an appropriately located organ may not increase the load of the heart, but reduce it.

Methods

The construction of the model is shown in Figure 1. An elastic tube (0.8 cm i.d., 1.1 cm o.d.) has a side branch (B) to which a small balloon (volume=1.5 cm^3) is connected via a hard tube (0.25 cm i.d., 0.4 cm o.d.). The length of the hard tube (X) was the experimental variable; it was varied from 2 to 100 cm. The pump (p) used to simulate the heart was a pulsatile one (Master-flex Digital unified drive from Cole-Parmer Instrument Co., Chicago). We measured the pulse with a pressure transducer (Validyne model DP103 with a frequency response of 0–1,000 Hz [Validyne Engineering Corp., Northridge, Calif.]) at position T marked in Figure 1.

This system is a simulation of the experiment performed on rats described previously.[7–9] The elas-

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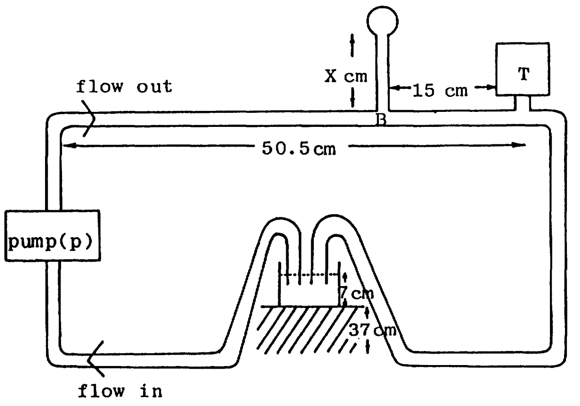

FIGURE 1. *The physical model that simulates a segment of the artery system is illustrated. The coupling resonance effect of the main elastic tube (0.8 cm i.d., 1.1 cm o.d.) with the side branch balloon (B) (balloon volume, 1.5 cm*^{3}*; hard tube, 0.25 cm i.d., 0.4 cm o.d.) was found. Water was pumped out by a pulsatile pump (p) from the water pool (water level, 7 cm), which was placed 37 cm higher than the tubing system. The side tube length (X) was varied from 2 to 100 cm; the corresponding pressure pulses were measured with a pressure transducer at T, which was 15 cm from the side branch B, or more than 50.5 cm from the pump.*

tic tube represents the aorta, and the small balloon simulates an organ attached to the aortic trunk.

Results

Pressure waves with the side branch balloon and without the side branch balloon (by blocking the side branch) were recorded. The mean pressure of the system was not changed before and after the blockage. We defined

$$C_0 = \frac{1}{\tau}\int_0^{\tau} (P - P_0)\, dt$$

where C_0 is the zeroth term of the Fourier series, τ is the period of one beat, and P_0 is the initial pressure of the beat, which is equivalent to the diastolic pressure. After some observation, we found that if we define C_0 this way, C_0 seemed to be a good indicator of the coupling condition. The recorded pressures were compared by their harmonic proportions (harmonic moduli normalized by their associated C_0). Changes in the difference in harmonic proportions of the two conditions as a function of the length of the side branch are illustrated in Figure 2. We analyzed the pressure above P_0 only. In other words, the traveling pressure wave similar to alternating voltage was analyzed, and the constant static voltage was ignored. This comparison helps us see the frequency responses of the two conditions.

The resonance model helped us understand the effect of the organs on the blood pressure wave. When the hard tube was short (X was small), the moduli of the first harmonic and C_0 tended to increase if the side branch was ligated, while those of the higher harmonics tended to decrease. There were some fluctuations in these moduli when X was changed; the variation curve usually became flat when $X > 30$ cm. This phenomenon may be understood through the electrical power line analogy of the vessel (artery).

Theory

From Noordergraaf,[10] we know that a vessel may be represented by an analogous electric circuit. The equivalent circuit of an artery tube with a side branch organ is obtained by the three steps shown in Figure 3.

The side branch with a balloon attached does not behave only as a pure reflection site; it may induce a coupled oscillation with the main tube as well. The coupling effect of the main distensible tube with the side balloon depends on the size, elasticity, and position of the balloon. When the balloon is at an appropriate distance from the main tube, the resonance property of the system enhances those harmonics in the resonance bands. This coupling will become weaker when the balloon distance X is large and the balloon will again become only a reflection site.

In an electric circuit as shown in step C of Figure 3, the impedance at frequencies $\omega_1 = 1/\sqrt{LC^*}$ and $\omega_2 = 1/\sqrt{L^*C}$ will be greatly reduced, and the ω_1 and ω_2 are called resonance frequencies of this circuit. Because of the existence of resistance in both resonance units, the ω_1 and ω_2 peaks will be broadened to two bands centered at ω_1 and ω_2.

The transmission of a viscoelastic tube can be drawn schematically (Figure 4, sketch 1).[11] (There is a cutoff in transmission at higher frequencies.) The side branch balloon introduces two resonance bands. With these two resonance bands, transmission will be increased by two broad peaks at ω_1 and ω_2. If ω_1 or ω_2 is at the higher frequency region (from our observations, ω_2 is probably at the higher frequency region), we will see that transmission at the higher frequencies is increased because of the resonance phenomenon (sketch 2).

At strong coupling, C^*, which was the compliance of the balloon, became part of the compliance of the large elastic tube. Therefore, some fluid would actually flow a round-trip through the small tube (to the balloon and then back to the large elastic tube).

The leakage resistance in C^* was about twice the resistance of the small hard tube. When the balloon was at resonance with the large elastic tube, it was easier for high-frequency components (around ω_2) of the pressure wave to travel along the main elastic tube without entering the small tube. Therefore, the increase of the resistance in this simulation was mainly in the low-frequency components, or small α (Figure 4, sketch 3).

Discussion

An organ such as a kidney or a group of arterioles behaves like the balloon. When the distance X is appropriate, the resonance effect helps the blood

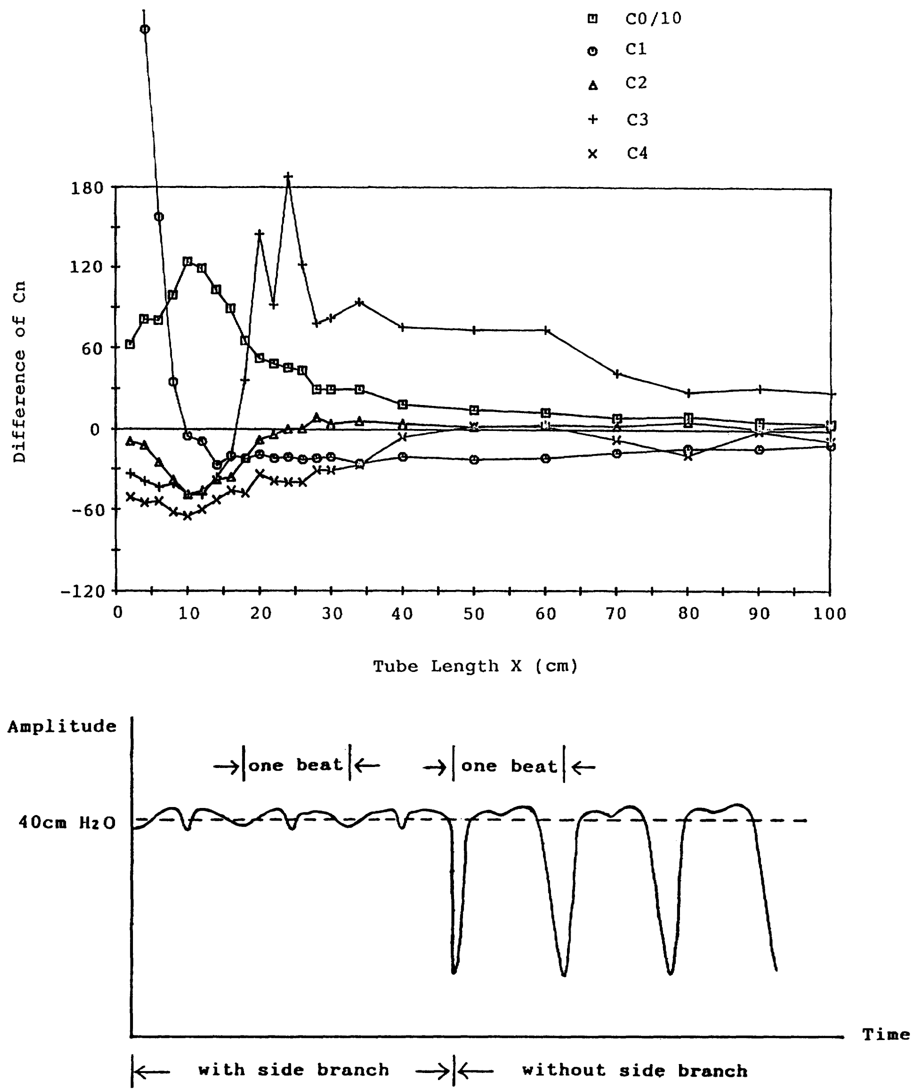

FIGURE 2. Top panel: *The changes in the difference in harmonic proportions with and without the side branch balloon as a function of the side branch tube length (X) are illustrated.* C_n *is the* nth *harmonic modulus normalized by the modulus of the* 0th *harmonic; that is,* $C_n = (n\text{th harmonic modulus}/0\text{th harmonic modulus}) \times 100\%$. *Difference of* $C_n = [(C_n \text{ without the side branch} - C_n \text{ with the side branch})/C_n \text{ with the side branch}] \times 100\%$. *With a side branch length less than 10 cm, the attached balloon is similar to the addition of a capacitance. It decreases the low-frequency* C_n *and increases the high-frequency* C_n*; at* $X=10$ *cm the effect is most profound.* Bottom panel: *The pulse shape for* $X=10$ *cm.*

enter the organ. It also amplifies the high-frequency blood pressure waves traveling along the main artery. If we tie the side branch toward the organ (balloon), some of the high-frequency components are reduced significantly. If we release it, the amplitudes of these high frequencies return to their normal values. This

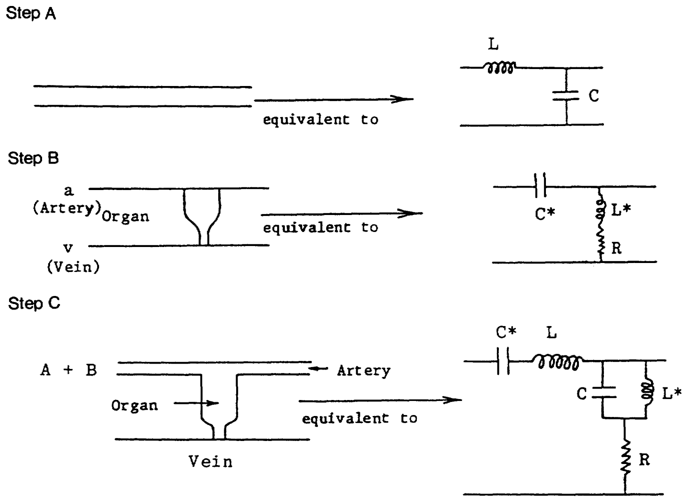

FIGURE 3. *Equivalent circuit of an artery tube with a side branch organ obtained by three steps. In step A, a uniform vessel behaves like a low-pass filter. A side branch organ can be treated in the same way (step B). Because of the property of linearity, the analogous representation of the artery tube with a side branch organ is a linear combination of the two (step C). L, equivalent inductance of the uniform vessel; C, equivalent capacitance of the uniform vessel; L*, equivalent inductance of the side branch organ; C*, equivalent capacitance of the side branch organ; R, resistance.*

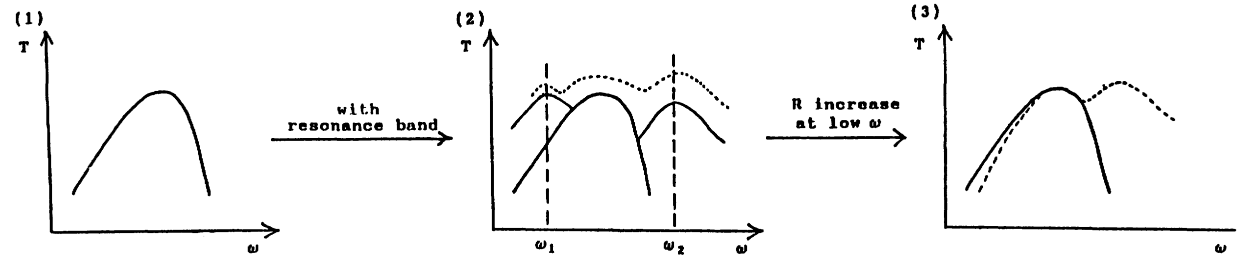

FIGURE 4. *Schematic representation of the transmission (T) of a viscoelastic tube (sketch 1). The side branch balloon introduces two resonance bands with peaks at frequencies ω_1 and ω_2 (sketch 2). An increase in resistance (R) was mainly in the low-frequency components (sketch 3).*

TABLE 1. Differences in Harmonic Proportions of Pressure Waves

Harmonic No.	Difference of C_n					
	Rat 1	Rat 2	Rat 3	Rat 4	Rat 5	Balloon
1	3.7	3.5	−4.5	0.5	0.9	−5.5
2	−49.4	−43.5	−45.0	−36.8	−12.2	−48.7
3	−64.9	−56.5	−52.5	−40.5	−21.2	−49.3
4	−77.9	−70.1	−73.2	−56.9	−25.6	−65.5
5	−82.2	−74.4	−73.9	−58.0	−22.7	−77.8
6	−81.6	−63.8	−62.8	−42.1	−32.0	−85.9
7	−80.9	−49.1	−55.7	−23.6	−5.7	−93.2
8	−71.4	−45.3	−53.6	−58.5	13.5	−98.1
9	−58.4	−32.7	−49.0	−42.5	0.5	−91.0
10	−73.6	62.8	28.4	−31.9	96.3	−95.5

Data for rats 1–5 are the differences before and during the clamping of the left renal artery. The last column is the difference with and without the side branch when the balloon is at a distance of X=10 cm. C_n, ratio of the *n*th harmonic amplitude to C_0; C_0, the zeroth term of the Fourier series. The rat data are from Reference 7; similar data can be found in Reference 8.

is the result we found in the rat kidney experiments and our balloon simulation (Table 1).

In the physical model simulation experiment, we found that variation of the static pressure applied to the system strongly affects the resonance condition. There was also an optimal distance between the attached balloon and the main tube. In this construction it was about 10 cm (Figure 2).

Resonance is very important to high-frequency blood pressure wave propagation; therefore, the diastolic pressure, the position of the attached organ, and the physical properties of the attached organ, all of which influence the resonance condition, are important factors in blood pressure wave propagation.

The effect from the ligation of the mesenteric artery may be explained by reflection, because no organs are closely attached to the aorta by the mesenteric artery. This is similar to a side branch with a large distance where resonance is not so important.

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